1) Si: $\underbrace{sen\theta}_{a} + \underbrace{csc\theta}_{b} = m$

$\underbrace{sen\theta}_{a} - \underbrace{csc\theta}_{b} = n$

Hallar:

$m^2 - n^2$

$(a+b)^2 - (a-b)^2$

$\cancel{a^2} + 2ab \cancel{+b^2} \cancel{-a^2} + 2ab \cancel{-b^2}$

$4ab$

$4(sen\theta \cdot csc\theta)$

$4(1) = 4$

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2) $sen\theta \cdot cos\theta = \frac{1}{9}$

$\frac{1}{csc\theta} \cdot \frac{1}{sec\theta} = \frac{1}{9}$

$\frac{1}{csc\theta \cdot sec\theta} = \frac{1}{9}$

$9^2 = (csc\theta \cdot sec\theta)^2$

$81 = csc^2\theta \cdot sec^2\theta$

Hallar:

$\sqrt[3]{sec^2\theta + csc^2\theta - 1}$

$\sqrt[3]{sec^2\theta \cdot csc^2\theta - 1}$

$\sqrt[3]{81-1} = \sqrt[3]{80} = 2\sqrt[3]{10}$

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3) Hallar A y B para que la expresión sea una identidad:

I) $\left(\frac{1 - senx\,cosx}{1 - cotx}\right)\left(\frac{sen^4x - cos^4x}{sen^3x + cos^3x}\right) = A\,senx$

$\left(\frac{1 - senx\,cosx}{1 - \frac{cosx}{senx}}\right)\left(\frac{\overbrace{(sen^2x + cos^2x)}^{1}\cancel{(senx+cosx)}(senx - cosx)}{\cancel{(senx+cosx)}(\underbrace{sen^2x - senx\,cosx + cos^2x}_{1})}\right) = A\,senx$

$\boxed{\frac{\cancel{(1 - senx\,cosx)}}{\frac{senx - cosx}{senx}}} \cdot \frac{(senx - cosx)}{\cancel{(1 - senx \cdot cosx)}} = A\,senx$

$\frac{senx}{\cancel{(senx - cosx)}} \cdot \cancel{(senx - cosx)} = A\,senx$

$1\,senx = A\,senx \quad \therefore A = 1$

$\underline{II}$) $\left(\dfrac{1+\operatorname{sen}x+\cos x}{\sqrt{B}}\right)^2=(1+\operatorname{sen}x)(1+\cos x)$

$$\frac{(1+\operatorname{sen}x+\cos x)^2}{B}=(1+\operatorname{sen}x)(1+\cos x)$$

$$(1+\operatorname{sen}x+\cos x)^2=B(1+\operatorname{sen}x)(1+\cos x)$$

*Por identidad auxiliar:

$$\underline{2}(1+\operatorname{sen}x)(1+\cos x)=\underline{B}(1+\operatorname{sen}x)(1+\cos x)$$

$$\therefore \underline{B=2}$$

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4) Si: $\sec\theta+\tan\theta=4$

Hallar:

$$M=15\cot\theta+17\cos\theta$$

se sabe que:

$$\sec^2\theta-\tan^2\theta=1$$

$$\underbrace{(\sec\theta+\tan\theta)}_{4}(\sec\theta-\tan\theta)=1$$

$$\sec\theta-\tan\theta=\frac{1}{4}$$

$\underline{I}$)

$$\sec\theta+\tan\theta=4 \quad +$$
$$\sec\theta-\tan\theta=\frac{1}{4}$$
$$2\sec\theta=\frac{17}{4}$$
$$\sec\theta=\frac{17}{8}$$
$$\cos\theta=\frac{8}{17}$$

$\underline{II}$)

$$\sec\theta+\tan\theta=4 \quad -$$
$$\sec\theta-\tan\theta=\frac{1}{4}$$
$$2\tan\theta=\frac{15}{4}$$
$$\tan\theta=\frac{15}{8}$$
$$\operatorname{ctg}\theta=\frac{8}{15}$$

$$M=\cancel{15}\cdot\frac{8}{\cancel{15}}+\cancel{17}\cdot\frac{8}{\cancel{17}}$$

$$M=8+8=\underline{16}$$

5) Si: $(\text{sen}\,\alpha + \cos\alpha)^2 = \left(\frac{2}{3}\right)^2$

$1 + 2\,\text{sen}\,\alpha\cos\alpha = \frac{4}{9}$

$2\,\text{sen}\,\alpha\cos\alpha = \frac{4}{9} - 1$

$(2\,\text{sen}\cos\alpha)^2 = \left(\frac{-5}{9}\right)^2$

$4\,\text{sen}^2\alpha\cos^2\alpha = \frac{25}{81}$

$2\,\text{sen}^2\alpha\cos^2\alpha = \frac{25}{162}$

Hallar:

$M = \sqrt{162(\text{sen}^4\alpha + \cos^4\alpha) + 7}$

$M = \sqrt{162(1 - 2\,\text{sen}^2\alpha\cos^2\alpha) + 7}$

$M = \sqrt{162\left(1 - \frac{25}{162}\right) + 7}$

$M = \sqrt{\cancel{162}\left(\frac{137}{\cancel{162}}\right) + 7}$

$M = \sqrt{137 + 7} = \sqrt{144} = \underline{12}$