

PRÁCTICA DIRIGIDA – LÍMITES – ÁLBEGRA – 5TO B SEC.

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Práctica dirigida - Límites

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① $\lim_{x \rightarrow 3/2} \frac{6x^2 + 7x - 3}{2x^2 + 11x + 12}$

$$\begin{array}{r} 6x^2 + 7x - 3 \\ 2x \quad + 3 \\ 3x \quad - 1 \end{array}$$

$$\frac{\cancel{(2x+3)}(3x-1)}{\cancel{(2x+3)}(x+4)} = \frac{3x-1}{x+4}$$

$$\begin{array}{r} 2x^2 + 11x + 12 \\ 2x \quad + 3 \\ x \quad + 4 \end{array}$$

$$\frac{3\left(\frac{3}{2}\right) - 1}{\frac{3}{2} + 4}$$

$$\frac{\frac{7}{2}}{\frac{11}{2}} = \frac{7}{11}$$

② $\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^3 - 7x^2 + 16x - 12}$

$$\frac{x^3 - 3x^2 + 4}{x^3 - 7x^2 + 16x - 12}$$

$$\frac{1 - 3 + 4}{1 - 7 + 16 - 12}$$

$$\frac{1 - 0 + 0}{1 - 0 + 0 - 0} = \frac{1}{1} = 1$$

③ $\lim_{x \rightarrow 2} \frac{3x - 6}{1 - \sqrt{4x-7}}$

$$\frac{3x - 6}{1 - \sqrt{4x-7}} \cdot \frac{(1 + \sqrt{4x-7})}{(1 + \sqrt{4x-7})}$$

$$\frac{3x - 6 (1 + \sqrt{4x-7})}{1^2 - (\sqrt{4x-7})^2}$$

$$\frac{3x - 6 (1 + \sqrt{4x-7})}{1 - 4x + 7}$$

$$\frac{\cancel{(x-2)}(3)(1 + \sqrt{4x-7})}{\cancel{(x-2)}(-4)}$$

$$\frac{3(1 + \sqrt{4x-7})}{-4}$$

$$\frac{-6}{-4} \rightarrow \frac{-3}{2}$$

$$\textcircled{02} \lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}}$$

$$\frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \cdot \frac{(1 + \sqrt{5-x})}{(1 + \sqrt{5-x})} \cdot \frac{(3 + \sqrt{5+x})}{(3 + \sqrt{5+x})}$$

$$\frac{(9 - 5 - x)(1 + \sqrt{5-x})}{(1 - 5 + x)(3 + \sqrt{5+x})}$$

$$\frac{-(-4+x)(1 + \sqrt{5-x})}{(-4+x)(3 + \sqrt{5+x})}$$

$$\rightarrow \frac{-(1+1)1}{3+3} = \frac{-2}{6} = \frac{-1}{3}$$

$$\textcircled{03} \begin{aligned} f(x) &= 2x^2 - 3 \\ g(x) &= x + 1 + a \end{aligned}$$

Halla "a" si: $\lim_{x \rightarrow 0} f(g(x)) = 5$

$$\textcircled{04} \lim_{x \rightarrow 0} \frac{|x|}{x} \text{ no existe} \rightarrow (V)$$

$$\lim_{x \rightarrow 3} \frac{x^3 + 3}{x + 3} = 27 \rightarrow \frac{3^3 + 3}{3 + 3} = \frac{30}{6} \quad (F)$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3} = \frac{1}{2} \rightarrow \frac{(x-2)(x-1)}{(x-3)(x-1)} \rightarrow \frac{-1}{-2} = \frac{1}{2} \quad (V)$$

07) $\lim_{x \rightarrow 1} f(x)$

$$f(x) = \begin{cases} 3x^2 + 2x, & \text{si } x \leq 1 \\ \sqrt{x-1} + 5, & \text{si } x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1} 3x^2 + 2x = 3 + 2 = 5$$

$$\lim_{x \rightarrow 1} \sqrt{x-1} + 5 = 0 + 5 = 5$$

08)

$$A = \lim_{x \rightarrow 8} \frac{\sqrt[3]{x^2 - 4} \sqrt[3]{x} + 4}{(x-8)^2}$$

$$\left(\frac{\sqrt[3]{x-2}}{(x-8)^2} \right)^2$$

$$\left(\frac{\sqrt[3]{8^2 + 2} \sqrt[3]{8} + 4}{4 + 4 + 4} \right)^2 \Rightarrow \frac{1}{144}$$

09) $\lim_{x \rightarrow 2} \left(\frac{12}{8-x^3} - \frac{1}{2-x} \right)$

$$\frac{12}{(2-x)(4+2x)} - \frac{1}{2-x}$$

10) $\lim_{x \rightarrow 3} \frac{P(x)}{Q(x)}$, si

$$P(x) = 2x^3 - 5x^2$$

$$Q(x) = 4x^3 - 13x^2$$

$$P(x) \quad \left| \quad \begin{matrix} 2 & -5 \\ 4 & -13 \end{matrix} \right.$$

3 6

$$10) \lim_{x \rightarrow 2} \left(\frac{12}{8-x^3} - \frac{1}{2-x} \right)$$

$$\frac{12}{(2-x)(4+2x+x^2)} - \frac{1}{2-x}$$

$$\frac{12 - (4+2x+x^2)}{(2-x)(4+2x+x^2)}$$

$$\frac{8-2x-x^2}{(2-x)(4+2x+x^2)}$$

$$\frac{(x+4)(x-2)}{(2-x)(4+2x+x^2)}$$

$$\frac{6}{12} \rightarrow \frac{1}{2}$$

$$11) \lim_{x \rightarrow 3} \frac{P(x)}{Q(x)}$$

$$P(x) = 2x^3 - 5x^2 - 2x - 3$$

$$Q(x) = 4x^3 - 13x^2 + 4x - 3$$

P(x)

2	-5	-2	-3
3	6	3	3
$2x^2$	$1x$	1	0

Q(x)

4	-13	4	-3
3	12	-3	3
$4x^2$	$-1x$	1	0

$$\frac{(2x^2+x+1)(x-3)}{(4x^2-x+1)(x-3)} \Rightarrow \frac{18+3+1}{36-3+1} = \frac{22}{34} = \frac{11}{17}$$

$$12) \lim_{x \rightarrow a}$$

$$\frac{x^2 - a^2}{2x^2 - ax - a^2}$$

$$\begin{array}{r} 2x^2 - ax - a^2 \\ 2x \quad -a \\ 2x \quad +a \end{array}$$

$$\frac{(x-a)(x+a)}{(2x+a)(x-a)}$$

$$\rightarrow \frac{2a}{3a} = \frac{2}{3}$$

$$\textcircled{15} \lim_{x \rightarrow a} \frac{x^2 - (a-1)x - a}{x^2 - (a-2)x - 2a}$$

$$\frac{x^2 - ax + x - a}{x^2 - 2x - 2a}$$

$$\frac{x(x-a) + 1(x-a)}{x(x-a) + 2(x-a)}$$

$$\frac{x+1}{x+2} \cdot \frac{(x-a)}{(x-a)} = \frac{x+1}{x+2}$$

$\textcircled{16}$

$$(17) \lim_{x \rightarrow m} \frac{x^2 - mx + 3x - 3m}{x - m} = m^2 - 27$$

$$\frac{x(x-m) + 3(x-m)}{x-m} = m^2 - 27$$

$$m+3 = m^2 - 27$$

$$m^2 + m - 30$$

$$m = 5$$

$$m = -6$$

$$m = -6$$

$$m = 5$$

$$(18) \lim_{x \rightarrow 1} \frac{x^3 - 2a^2x + ax^2}{2ax + x^2} = 2a - 5 \quad a > 0$$

$$\frac{x^3 - 2a^2x + ax^2}{2ax + x^2} = 2a - 5$$

$$\frac{-(2a^2 - ax - x^2)}{2a + x^2} = 2a - 5$$

$$\frac{-(2a+1)(a-1)}{2a+1}$$

$$1 - a = 2a - 5$$

$$6 = 3a$$

$$2 = a$$

$$(19) \lim_{x \rightarrow 1} f(x) ; \lim_{x \rightarrow 4} f(x)$$

$$f(x) = \begin{cases} x^2 & \text{si } x < 1 \\ x & \text{si } 1 < x < 4 \\ 4-x & \text{si } x > 4 \end{cases}$$

$$\lim_{x \rightarrow 1} x^2 = 1$$

$$\lim_{x \rightarrow 4} x^2 = 16$$

$$\lim_{x \rightarrow 1} x = 1$$

$$\lim_{x \rightarrow 4} x = 4$$

$$\lim_{x \rightarrow 1} 4-x = 3$$

$$\lim_{x \rightarrow 4} 4-x = 0$$