(13) $\lim \dfrac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1} = \lim\limits_{x \to 1} \dfrac{x^{1/3} - 1}{x^{1/4} - 1}$

Derivando

$$= \lim_{x \to 1} \frac{\frac{1}{3} x^{-2/3}}{\frac{1}{4} x^{-3/4}}$$

$$= \frac{4}{3} \lim_{x \to 1} \frac{x^{3/4}}{x^{2/3}}$$

$$= \frac{4}{3} \frac{(1)^{3/4}}{(1)^{2/3}} = \frac{4}{3} \frac{(1)}{(1)} = \frac{4}{3}$$

(14) Si $A = \lim\limits_{x \to 1} \left(\dfrac{2}{x - 1} + \dfrac{3}{1 - x^3} \right)$

$B = \lim\limits_{x \to 1} \dfrac{\sqrt[3]{x - 2} - \sqrt[2]{1 - x + x^2}}{1 - x^2}$

$$\frac{A + B}{A - B} =$$

$$A = \lim_{x \to 1} \frac{2}{(x+1)(x-1)} + \frac{3}{(1-x)(1+x+x^2)}$$

$$= \lim_{x \to 1} \frac{2}{(x+1)(x-1} - \frac{3}{(x-1)(x^2+x+1)}$$

$$= \lim_{x \to 1} \frac{2(x^2+x+1 - 3(x+1)}{(x+1)(x-1)(x^2+x+1)}$$

$$= \lim_{x \to 1} \frac{2x^2 + 2x + 2 - 3x - 3}{(x+1)(x-1)(x^2+x+1}$$

$$= \lim_{x \to 1} \frac{2x^2 - x - 1}{(x+1)(x-1)(x^2+x+1)} = \lim_{x \to 1} \frac{(2x+1)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}(x^2+x}$$

$$= \lim_{x \to 1} \frac{2x+1}{(x+1(x^2+x+1)}$$

$$= \frac{2+1}{(1+1)(1+1+1)} = \frac{3}{2(3)} = \frac{1}{2}$$

Justus

$$B = \lim_{x \to 1} \frac{\sqrt[3]{x-2} + \sqrt[3]{1-x+x^2}}{1-x^2}$$

$$= \lim_{x \to 1} \frac{(\sqrt[3]{x-2}+1 + (\sqrt[3]{1-x+x^2}-1)}{(1-x)(1+x)}$$

$$= \lim_{x \to 1} \left[\frac{\sqrt[3]{x-2}+1}{(1-x)(1+x} \quad \frac{+\sqrt{1-x+x^2}-1}{(1-x)(1+x} \right]$$

$$= \lim_{x \to 1} \frac{x-2+1}{(1-x)(1+x)(\sqrt[3]{x-2})^2 - \sqrt[3]{x-2}+1} + \frac{1-x+x^2-1}{(1-x)(1+x)(\sqrt{1-x+x^2}+1)}$$

$$= \lim_{x \to 1} \frac{1}{(1+x)(\sqrt[3]{x-2})^2 - \sqrt[3]{x-2}+1)} + \frac{x^2-x}{(1-x)(1+x)(\sqrt{1-x+x^2}+1)}$$

$$= \lim_{x \to 1} \left[\frac{1}{(1+x)(\sqrt[3]{(x-2)})^2 - \sqrt[3]{x-2}+1} + \frac{x(x-1)}{(1-x)(1+x)(\sqrt{1-x+x^2}+1)} \right]$$

$$= \lim_{x \to 1} \frac{1}{(1+1)(\sqrt[3]{1} - \sqrt[3]{-1}+1} + \frac{1}{2(\sqrt{1-1+1}+1)}$$

$$= \lim_{x \to 1} \frac{1}{2(1+1+1)} + \frac{1}{2(1+1)}$$

$$= \frac{1}{6} + \frac{1}{4} = \frac{2+3}{12} = \frac{5}{12}$$

$$\frac{A+B}{A-B} = \frac{\frac{1}{2}+\frac{5}{12}}{\frac{1}{2}-\frac{5}{12}} = \frac{\frac{6+5}{\cancel{12}}}{\frac{6-5}{\cancel{12}}}$$

$$= \frac{11}{1} = 11$$

15 $\lim_{x \to a} \dfrac{x^2 - (a-1)x - a}{x^2 - (a-2)x - 2a}$

$= \lim_{x \to a} \dfrac{\cancel{(x-a)}(x+1)}{\cancel{(x-a)}(x+2)}$

$= \dfrac{a+1}{a+2}$

16 * $\lim_{x \to 2} \dfrac{x^4 + x^3 - 24}{x^2 - 4}$

$= \lim_{x \to 2} \dfrac{\cancel{(x-2)}(x^3 + 3x^2 + 6x + 12)}{\cancel{(x-2)}(x+2)}$

$= \dfrac{8+12+12+12}{4} = \dfrac{20+24}{4} = \dfrac{44}{4} = 11$

* $\lim_{x \to 1} \dfrac{x^3 + x^2 - 5x + 3}{x^3 + 2x^2 - 7x + 4}$

$= \lim_{x \to 1} \dfrac{\cancel{(x-1)}(x^2 + 2x - 3)}{\cancel{(x-1)}(x^2 + 3x - 4)}$

$= \lim_{x \to 1} \dfrac{(x+3)\cancel{(x-1)}}{(x+4)\cancel{(x-1)}} = \dfrac{1+3}{1+4} = \dfrac{4}{5}$

* $\lim_{x \to 1} \dfrac{5x^2 + 3x^5 - 8}{7x^4 - 4x - 3}$

	3	0	0	5	0	-8
1	↓	3	3	3	8	8
	3	3	3	8	8	$\boxed{0}$

	7	0	0	-4	-3
1	↓	7	7	7	3
	7	7	7	3	$\boxed{0}$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(3x^4+3x^3+3x^2+8x+8)}{\cancel{(x-1)}(7x^3+7x^2+7x+3)}$$

$$= \frac{3+3+3+8+8}{7+7+7+3} = \frac{25}{24}$$

* $\lim_{x \to 3} \frac{x^3+6x^2+9x}{x^3+5x^2+3x-9}$

Numerador

$$x^3+6x^2+9x = x(x^2+6x+9)$$
$$= x(x+3)^2$$

Denominador

$$\begin{array}{r|rrrr} & 1 & 5 & 3 & -9 \\ -3 & \downarrow & -3 & -6 & 9 \\ \hline & 1 & 2 & -3 & \boxed{0} \end{array}$$

$$= (x+3)(x^2+2x-3)$$
$$= (x+3)(x+3)(x-1)$$

$$= \lim \frac{x\cancel{(x+3)^2}}{\cancel{(x+3)^2}(x-1)}$$

$$= \frac{3}{3-1} = \frac{3}{2}$$

* $\lim_{x \to 0} \frac{\sqrt{1+x^2}-1}{x^2}$

$$= \lim_{x \to 0} \frac{(\sqrt{1+x^2}-1)(\sqrt{1+x^2}+1)}{x^2(\sqrt{1+x^2}+1)}$$

$$= \lim_{x \to 0} \frac{\cancel{1}+x^2-\cancel{1}}{x^2(\sqrt{1+x^2}+1)}$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{1+x^2}+1)} = \lim_{x \to 0} \frac{1}{\sqrt{1+x^2}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

* $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$= \lim\limits_{x \to 0} \dfrac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim\limits_{x \to 0} \dfrac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{x(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim \dfrac{1+x-(1-x)}{x\sqrt{1+x}+\sqrt{1-x}}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{1}+x-\cancel{1}+x}{x(\sqrt{1+x}+\sqrt{1-x}}$

$= \lim\limits_{x \to 0} \dfrac{2\cancel{x}}{\cancel{x}(\sqrt{1+x}+\sqrt{1-x})}$

$= \lim\limits_{x \to 0} \dfrac{2}{\sqrt{1+x}+\sqrt{1-x}}$

$= \dfrac{2}{\sqrt{1}+\sqrt{1}} = \dfrac{2}{1+1} = \dfrac{2}{2} = 1$

* $\lim\limits_{x \to 5} \dfrac{\sqrt{x-4}-\sqrt{3x-14}}{x-5}$

$= \lim\limits_{x \to 5} \dfrac{\sqrt{x-4}-\sqrt{3x-14}}{(x-5)} \dfrac{\sqrt{x-4}+\sqrt{3x-14}}{(\sqrt{x-4}+\sqrt{3x-14}}$

$= \lim\limits_{x \to 5} \dfrac{(\sqrt{x-4})^2-(\sqrt{3x-14})^2}{(x-5)(\sqrt{x-4}+\sqrt{3x-14}}$

$= \lim\limits_{x \to 5} \dfrac{x-4\ (3x-14)}{(x-5\ (\sqrt{x-4}+\sqrt{3x-14}}$

$= \lim\limits_{x \to 5} \dfrac{-2x+10}{(x-5)(\sqrt{x-4}+\sqrt{3x-14}}$

$$= \lim_{x \to 5} \frac{-2\cancel{(x-5)}}{\cancel{(x-5)}(\sqrt{x-4} + \sqrt{3x-14})}$$

$$= \lim_{x \to 5} \frac{-2}{\sqrt{x-4}\sqrt{3x-14}}$$

$$= \frac{-2}{\sqrt{1} + \sqrt{1}} = \frac{-2}{1+1} = \frac{-2}{2} = -1$$

* $$\lim_{x \to 0} \frac{\sqrt[3]{8+x} - 2}{x} = \lim_{x \to 0} \frac{(\sqrt[3]{8+x} - 2)(\sqrt[3]{(8+x)^2} + 2\sqrt[3]{8+x} + (2)^2)}{x(\sqrt[3]{(8+x)^2} + 2\sqrt[3]{8+x} + 4)}$$

$$= \lim_{x \to 0} = \frac{8 + x - (2)^3}{x(\sqrt[3]{8+x})^2 + 2\sqrt[3]{8+x+4})}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt[3]{8+x})^2 + 2\sqrt[3]{8+x} + 4)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt[3]{(8+x)^2} + 2\sqrt[3]{8+x} + 4}$$

$$= \frac{1}{\sqrt[3]{8^2} + 2\sqrt[3]{8} + 4}$$

$$= \frac{1}{4+4+4}$$

$$= \frac{1}{12}$$