* $$\lim_{x \to 1} \frac{1-x^2}{(1+ax)^2-(a+x)^2}, \quad a>0$$

$$= \lim_{x \to 1} \frac{(1-x)(1+x)}{(1+ax+a+x)(1+ax-a-x)}$$

$$= \lim_{x \to 1} \frac{\cancel{(1-x)}\cancel{(1+x)}}{\cancel{(1+x)}(1+a)\,[\cancel{(1-x)}(1-a)]}$$

$$= \lim_{x \to 1} \frac{1}{(1+a)(1-a)}$$

$$= \frac{1}{1-a^2}$$

17 $$\lim_{x \to m} \frac{x^2-mx+3x-3m}{x-m} = m^2-27$$

$$\lim_{x \to m} \frac{x(x+3)-m(x+3)}{x-m} = m^2-27$$

$$= \lim_{x \to m} \frac{(x+3)\cancel{(x-m)}}{\cancel{x-m}} = m^2-27$$

$$= m+3 = m^2-27$$

$$m^2-m-30=0$$

$$(m-6)(m+5)=0$$

$$\begin{cases} m=6 \\ m=-5 \end{cases}$$

(18) Hallar el valor de "a"

$a > 0$, sabiendo que

$$\lim_{x \to 1} \frac{x^3 - 2a^2x + ax^2}{2ax + x^2} = 2a - 5$$

$$(1)^3 - 2a^2(1) + a(1)^2 = 2a - 5$$

$$\frac{1 - 2a^2 + a}{2a + 1} = 2a - 5$$

$$-2a^2 + a + 1 = (2a - 5)(2a + 1)$$
$$-2a^2 + a + 1 = 4a^2 + 2a - 10a - 5$$
$$-2a^2 + a + 1 = 4a^2 - 8a - 5$$
$$-2a^2 + a + 1 = 4a^2 + 8a + 5 = 0$$
$$-6a + 9a + 6 = 0$$
$$6a^2 + 9a - 6 = 0$$
$$2a^2 - 3a - 2 = 0$$
$$(2a + 1)(a - 2) = 0$$
$$\boxed{a = 2} \text{ Respuesta}$$
$$a = 1/2 \text{ (NO)}$$

(19) $\lim_{x \to 1}$; $\lim_{x \to 4} f(x)$

$$f(x) \begin{cases} x^2, & \text{si } x < 1 \\ x, & \text{si } 1 < x < 4 \\ 4 - x, & \text{si } x > 4 \end{cases}$$

$$\lim_{x \to 1^+} f(x) \lim_{x \to 1} x = 1$$

$$\lim_{x \to 1} f(x) \lim x^2 = (1)^2 = 1$$

$$\therefore \lim_{x \to 1} f(x) = 1$$

$$\lim_{x \to 4^+} = \lim_{x \to 4} (4-x) = 4-4 = 0$$

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} x = 4$$

$$\therefore \lim_{x \to 4} f(x) \text{ no existe}$$

20 Calcular si existen

$\lim f(x)$, donde

$$\lim_{x \to 5} f(x) \begin{cases} \dfrac{x-5}{1-\sqrt{x-4}} & ; \text{ si } x \geq 5 \\[2ex] \dfrac{x^2-1x+35}{x-5} & ; \text{ si } x < 5 \end{cases}$$

$$\lim_{x \to 5^+} f(x) \lim_{x \to 5} \frac{x-5}{1-\sqrt{x-4}}$$

$$= \lim_{x \to 5} \frac{(x-5)(1+\sqrt{x-4})}{(1)^2-(x-4)}$$

$$= \lim_{x \to 5} \frac{(x-5)(1+\sqrt{x-4})}{1-x+4}$$

$$= \lim_{x \to 5} \frac{\cancel{(x-5)}(1+\sqrt{x-4})}{\cancel{5-x}}$$

$$= -\lim_{x \to 5} (1+\sqrt{x-4})$$

$$= -(1+1) = -2$$

$$f = (x) \lim_{x \to 5} \frac{x^2 - 12x + 35}{x - 5} \; ; \; x < 5$$

$$= \lim_{x \to 5} \frac{(x-7)\cancel{(x-5)}}{\cancel{x-5}}$$

$$= 5 - 7$$

$$= -2 = -2$$

$$\therefore \lim_{x \to 5} f(x) = -2$$

(21) Calcular si existen $\lim_{x \to 1} f(x)$ donde

$$f(x) \begin{cases} \dfrac{1-\sqrt{x}}{1-\sqrt[3]{x}} , & x > 1 \\[2ex] \dfrac{x^2 - \frac{x}{2} - \frac{1}{2}}{x-1} , & x < 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{1-\sqrt{x}}{1-\sqrt[3]{x}}$$

$$= \lim_{x \to 1} \frac{[(1-\sqrt{x})(1+\sqrt{x})][1+\sqrt[3]{x}+\sqrt[3]{x^2}]}{[(1-\sqrt[3]{x})(1+\sqrt[3]{x}+\sqrt[3]{x^2})][1+\sqrt{x})}$$

$$= \lim_{x \to 1} \frac{\cancel{(1-x)}\,[1-\sqrt[3]{x}+\sqrt[3]{x^2}]}{\cancel{(1-x)}\,(1+\sqrt{x}}$$

$$= \frac{1+1+1}{1+1} = \frac{3}{2}$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - \frac{x}{2} - \frac{1}{2}}{x - 1}$$

$$= \lim_{x \to 1} \frac{2x^2 - x - 1}{2(x-1)}$$

$$= \lim_{x \to 1} \frac{(2x+1)\cancel{(x-1)}}{2\cancel{(x-1)}}$$

$$= \frac{2+1}{2} = \frac{3}{2}$$

$$\therefore \lim_{x \to 1} f(x)\ \frac{3}{2}$$

(22)

$$f(x) = \begin{cases} ax^2 + bx + 1 ; & x \leq 1 \\ 2ax - b ; & 1 < x \leq 2 \\ x + 1 ; & x > 2 \end{cases}$$

Hallar a y b Para que existan los límites de $f(x)$ en $x = 1$ y $x = 2$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} (2ax - b) = 2a - b$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (ax^2 + bx + 1) = a + b + 1$$

$$\therefore 2a - b = a + b + 1$$

$$\boxed{a = 2b + 1}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (x + 1) = 2 + 1 = 3$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2ax - b)$$

$$= 4a - b$$

$$\boxed{4a - b = 3}$$

$$4(2b+1)-b=3$$

$$8b+4-b=3$$

$$7b=-1$$

$$\boxed{b=-1/7}$$

$$a=2b+1$$

$$a=2\left(-\frac{1}{7}\right)+1$$

$$a=-\frac{2}{7}+1$$

$$\boxed{a=\frac{5}{7}}$$